

## Analysis of the Design Parameters for a Lightfield Near-eye Display Based on a Pinhole Array

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(Received December 23, 2019 : revised March 10, 2020 : accepted March 11, 2020)

With the increasing demand for head-mounted display applications, the image quality provided by a near-eye display device is a key factor in satisfying the consumer. Among various techniques to realize a near-eye display that has a thinner volume than the working distance of a human eye, a lightfield image-generation method based on a pinhole array is attracting much attention, with its simple and thin structure. In this paper, we propose a numerical analysis of the visual parameters and verifications with computational reconstruction.

**Keywords :** Near eye display, Lightfield image generation, Design parameter

**OCIS codes :** (100.6890) Three-dimensional image processing; (110.2990) Image formation theory; (110.3000) Image quality assessment

### I. INTRODUCTION

Nowadays, augmented reality (AR) and virtual reality (VR) head-mounted displays (HMDs) open a new era of visual experiences. Though the objects of an AR and a VR HMD differ from each other, there is a common point between them: The observer has to wear a near-eye display (NED) that provides the visual information to the eye directly. Since the NED technique is related to a thinner structure and a lighter weight, it is essential for realizing a practical consumer product. Among various techniques, a lightfield method based on a pinhole or a pinlight array is regarded as the simplest and most practical approach to delivering the blur-free image information to the observer's retinal surface, through multiple pinholes [1-6].

However, a numerical analysis of the relation between visual parameters and system parameters has not been provided yet, though several proposals to use a pinhole array to realize a NED have been introduced previously [1, 7]. Since it is useless to implement vanishingly small pinholes, due to the extremely low optical efficiency and image degradation from diffraction effects, the pinhole or

pinlight array must have a practical diameter of several millimeters, and is expected to provide an overlapping combination of slightly blurred retinal images of pixels on the display device. Therefore, the image quality of the observed image is dependent on the angular size of the blurred pixels, and the overlap ratio of the retinal images. Moreover, the pupil diameter of the observer, which will vary with the luminance of the displayed content, also affects the reconstruction of the retinal images. In this paper, we analyze the visual parameters of angular retinal resolution, overlap ratio of retinal elemental images, and the effect of varying pupil diameter of the observer. Verification of the theory with computational reconstruction is also provided.

### II. ANALYSES AND COMPUTATIONAL RECONSTRUCTION

Figure 1 shows the principles and system parameters of a pinhole-array-based NED with a gap  $D_{ap}$  between the display panel and the pinhole array. The NED projects the elemental images of length  $d_i$  to the retinal surface of an

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eyeball of diameter  $d_{eye}$ , through a corresponding pinhole aperture of diameter  $d_p$ . The eye's lens is far from the pinhole array, by a distance  $D_{pp}$ , and is assumed to have aperture of diameter  $d_{pupil}$  and a focal length of  $f$ .

Since the retinal elemental images of diameter  $d$  overlap on the retinal surface with a gap of  $\beta$ , which is the distance between the centers of each retinal elemental image, the overlap ratio  $\alpha$  between the retinal elemental images can be calculated as follows, where  $g_e$  and  $g_{pp}$  are the gap between each elemental image and each pinhole respectively.

$$d = d_{pupil} \left( 1 - d_{eye} \left( \frac{1}{f} - \frac{d_p + d_{pupil}}{d_{pupil} D_{pp}} \right) \right), \quad (1)$$

$$\beta = \frac{d_{eye} (d_i + g_e)}{D_{dp} + D_{pp}}, \quad (2)$$

$$\alpha = \frac{d - \beta}{d} = 1 - \frac{\beta}{d} = 1 - \frac{d_{eye} D_{pp} f (d_i + g_e)}{(D_{dp} + D_{pp}) (D_{pp} d_{pupil} (f - d_{eye}) + d_{eye} f (d_p + d_{pupil}))}. \quad (3)$$

In Eq. (3), the overlap ratio  $\alpha$  will have a negative value when  $d < \beta$ , which means there are empty spaces between the retinal elemental images in the horizontal or vertical directions.

To avoid interference between elemental images, and to be able to use a panel, the gap  $g_{pp}$  between pinholes must satisfy the following Eq. (5), according to Fig. 2.

$$g_{pp} : d_{pupil} = D_{dp} : D_{dp} + D_{pp}, \quad (4)$$

$$g_{pp} = \frac{d_{pupil} D_{dp}}{D_{dp} + D_{pp}}. \quad (5)$$

Then, we can also calculate the retinal angular resolution  $A_R$ , in units of pixels per degree (PPD), from the angular

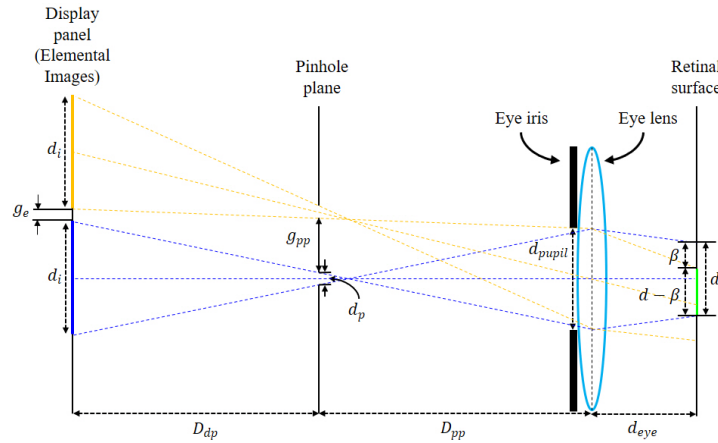


FIG. 1. The basic principles and parameters of a pinhole-array-based NED.

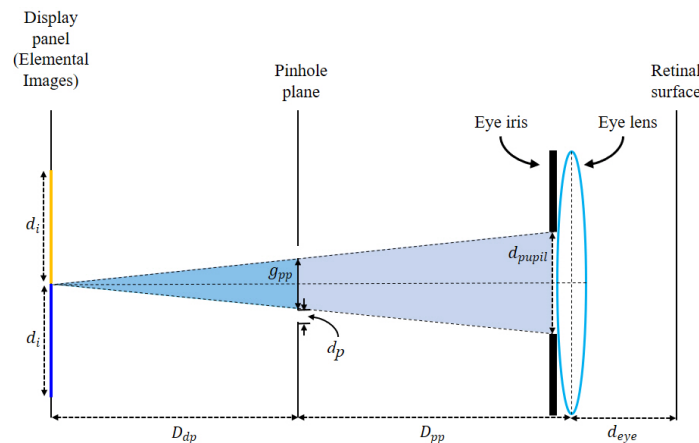
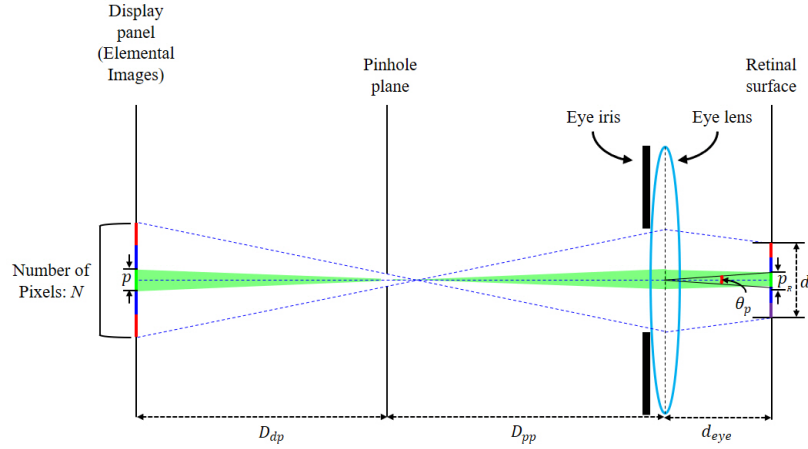
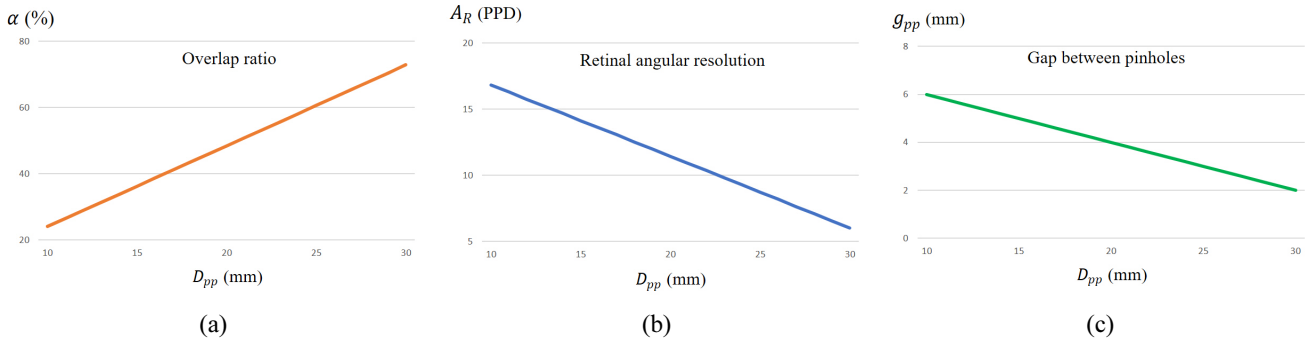


FIG. 2. Calculation of the gap  $g_{pp}$  between pinholes.

FIG. 3. Calculation of the angular size  $\theta_p$  of a single pixel on the retinal surface.FIG. 4. The relation between the distance  $D_{pp}$  from the pinhole array to the eye's pupil: (a) the overlap ratio, (b) the retinal angular resolution, and (c) the gap between pinholes.

size  $\theta_p$  of a retinal image of a single pixel comprising the elemental image. If we assume that the pixel width of the display panel is  $p$  as shown in Fig. 3, we can expect that each elemental image is composed of a number of pixels  $N$  and can calculate  $\theta_p$  as follows, using the retinal pixel width  $p_R$  and  $d$ .

$$N = \frac{d_i}{p}, \quad (6)$$

$$p_R = \frac{d}{N} = \frac{pd}{d_i}, \quad (7)$$

$$\theta_p = 2\arctan\left(\frac{p_R}{2d_{eye}}\right) = 2\arctan\left(\frac{pd}{2d_i d_{eye}}\right). \quad (8)$$

Using Eqs. (2) and (9), we can derive the retinal angular resolution  $A_R$  as follows:

$$A_R = \frac{1}{\theta_p} = \left[ 2\arctan\left(\frac{p}{2d_i} d_{pupil} \left( \frac{1}{d_{eye}} - \frac{1}{f} + \frac{d_p + d_{pupil}}{d_{pupil} D_{pp}} \right) \right) \right]^{-1}. \quad (9)$$

TABLE 1. Example values of viewing parameters for generation of elemental images

Pupil diameter of the eye $d_{pupil}$	8 mm
Focal length of the eye's lens $f$	21.8 mm
Aperture diameter of pinhole $d_p$	0.2 mm
Pixel width of the display panel $p$	0.031 mm
Eyeball diameter $d_{eye}$	22 mm

From the analysis above, we can expect that the overlap ratio  $\alpha$  and retinal angular resolution  $A_R$  have a tradeoff relation, as shown in Figs. 4(a) and 4(b), when the volume of the NED system is fixed. Also, the gap  $g_{pp}$  between pinholes has an inversely proportional relation with the overlap ratio, as shown in Fig. 4(c). In Fig. 4, the length of the NED system  $D_{dp} + D_{pp}$  is assumed to be 40 mm, with  $D_{pp}$  varying from 10 to 30 mm ( $D_{dp}$ : 30 mm - 10 mm). The viewing parameters are shown in Table 1.

From the analyses above, we can expect that it would be better to design the gap  $g_{pp}$  between the pinholes larger and the overlap ratio smaller by reducing the distance  $D_{pp}$  between the pinhole array and the eye's pupil, if we focus

on increasing the retinal angular resolution for a fixed length of NED. Figure 5 shows a comparison of the computationally reconstructed views for different values of

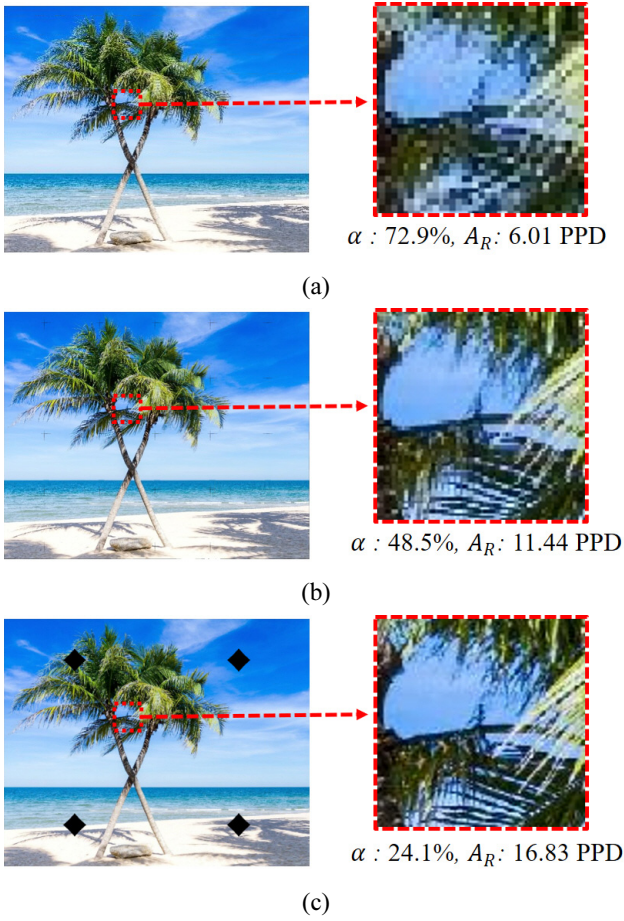


FIG. 5. Comparison of reconstructed views with different viewing parameters: (a)  $\alpha = 72.9\%$ ,  $A_R = 6.01$  PPD ( $D_{dp} = 10$  mm,  $D_{pp} = 30$  mm), (b)  $\alpha = 48.5\%$ ,  $A_R = 11.44$  PPD ( $D_{dp} = 20$  mm,  $D_{pp} = 20$  mm), and (c)  $\alpha = 24.1\%$ ,  $A_R = 16.83$  PPD ( $D_{dp} = 30$  mm,  $D_{pp} = 10$  mm).

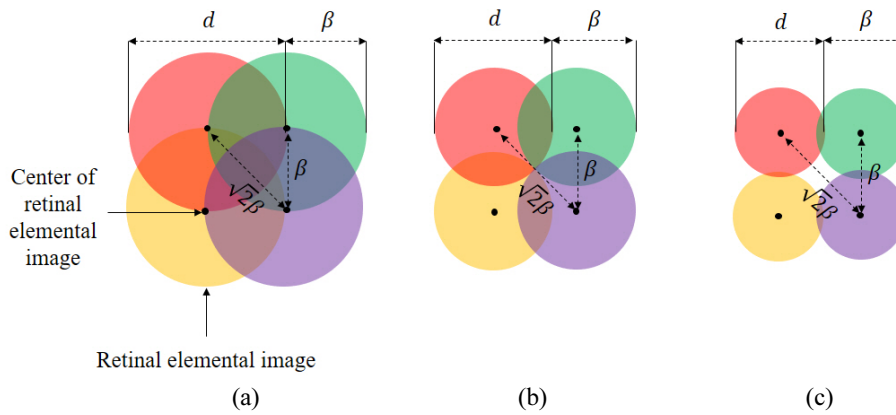


FIG. 6. Combinations of overlapping retinal elemental images projected by a square arrangement of pinhole apertures: (a)  $d > \sqrt{2}\beta$ , (b)  $d = \sqrt{2}\beta$  (critical condition), (c)  $d < \sqrt{2}\beta$ .

$D_{pp}$ , which verifies that the image quality of the reconstructed view is enhanced as the retinal angular resolution  $A_R$  becomes larger when  $D_{pp}$  decreases, for a fixed length of NED.

However, there is another factor, the critical overlap ratio  $\alpha_C$ , which we should also consider when designing the viewing parameters and generating the elemental images, to prevent empty spaces between retinal elemental images, as seen in Fig. 5(c). The critical overlap ratio is related to the geometry of the arrangement of pinhole apertures. Figure 6 shows conditions for overlapping images for different diameters of each retinal elemental image. If we assume a square arrangement of pinhole apertures, as shown in Fig. 6, the diameter of the retinal elemental image  $d$  should be larger than the diagonal distance  $\sqrt{2}\beta$  between centers of opposite elemental images on the retinal plane, to avoid empty spaces. Therefore, the critical overlap ratio  $\alpha_C$  can be derived as follows, for a square arrangement of elemental images:

$$d \geq \sqrt{2}\beta = \sqrt{2}d(1-\alpha), \quad (10)$$

$$\alpha \geq 1 - \frac{1}{\sqrt{2}} = \alpha_C \cong 0.293. \quad (11)$$

Therefore, when designing the viewing parameters of a NED and generating the elemental images, it is required to keep the overlap ratio  $\alpha$  greater than  $\alpha_C$ , to prevent any empty spaces between the retinal elemental images. Moreover, in real use, even though the overlap ratio  $\alpha$  may be greater than the critical value  $\alpha_C$ , there is another reason to increase  $\alpha$  still more: the variances of a real eye's pupil diameter  $d'_{pupil}$ . According to previous research, it is expected that the real pupil diameter of an observer will vary from around 3 to 8 mm, when the maximum luminance of the pinhole-based NED is assumed to be 100 cd/m<sup>2</sup> [8]. Thus it is also necessary to analyze how the real  $d'_{pupil}$  and  $D_{pp}$  affect reconstruction of the observed view on the retinal plane of a real user.

Figures 7(a) and 7(b) show what happens when the real pupil diameter  $d'_{pupil}$  is larger than the design parameter  $d_{pupil}$ . In that case, a circular area with larger diameter than that of designed elemental image will be projected

through each pinhole aperture, and flipped views caused by the neighboring elemental images can be presented to the observer, as shown in Figs. 7(a) and 7(b). Thus it is recommended to set the design parameter  $d_{pupil}$  larger than

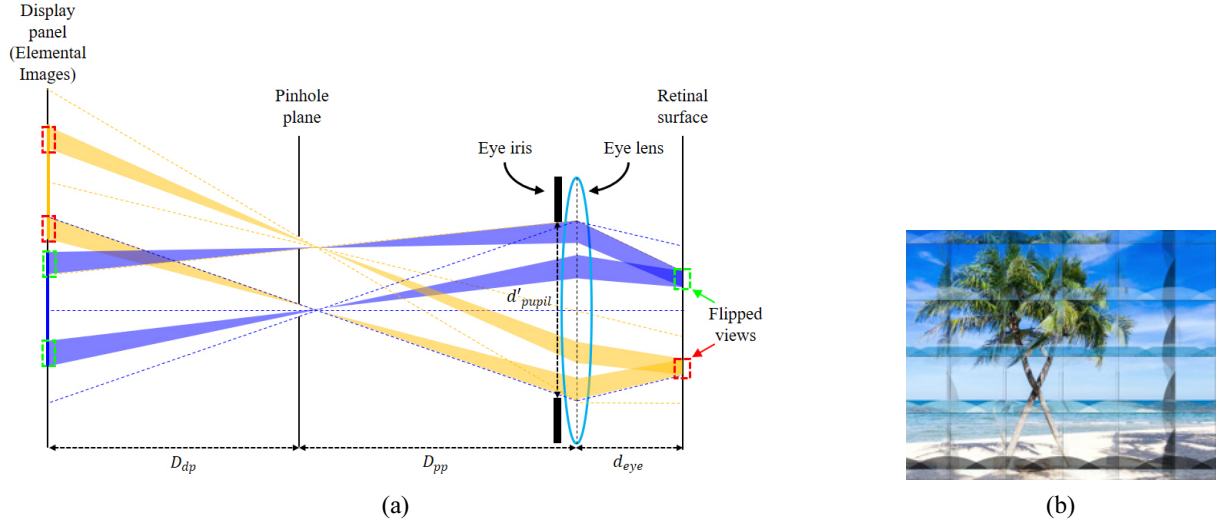


FIG. 7. Effect of larger real pupil diameter when  $d_{pupil} = 8$  mm: (a) wrong projection of elemental images, causing flipped views, and (b) reconstructed image with flipped views, when  $d'_{pupil} = 10$  mm,  $d_{pupil} = 8$  mm,  $D_{dp} = 20$  mm, and  $D_{pp} = 20$  mm.

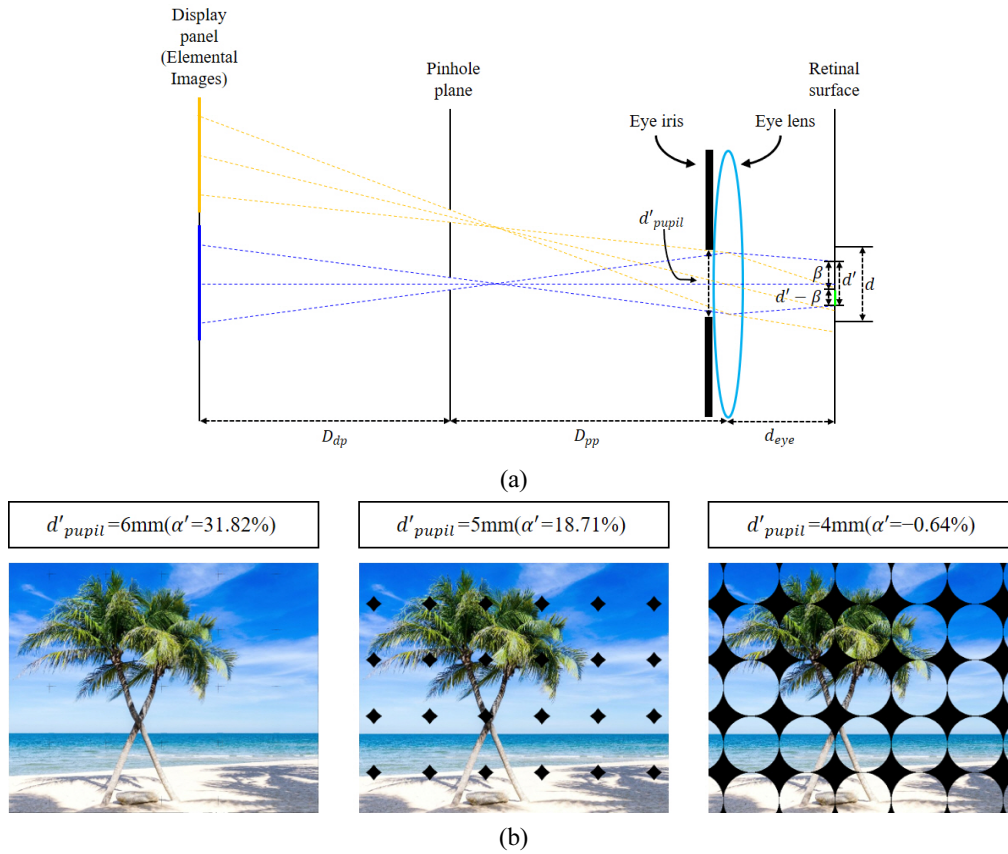


FIG. 8. Effect of a smaller real pupil diameter when  $d_{pupil} = 8$  mm: (a) partial projection of elemental images, causing the problem of empty spaces, and (b) reconstructed views with varying diameter  $d'_{pupil}$  of the real pupil, when  $D_{dp} = 20$  mm, and  $D_{pp} = 20$  mm.



the maximum diameter of a real pupil (typically 8 mm) [8], to prevent those flipped views.

In contrast, if  $d'_{pupil}$  is smaller than  $d_{pupil}$ , only a part of the designed elemental image will be projected to the retinal surface of the observer, and the real overlap ratio  $\alpha'$  will become smaller than the designed overlap ratio  $\alpha$ , as shown in Fig. 8(a). Then, when  $\alpha'$  is smaller than  $\alpha_C$  in Eq. (12), the problem of empty spaces between the projected retinal elemental images can occur, due to the unprojected areas of elemental images seen in Fig. 8(b), even though  $\alpha'$  is still positive. To prevent that problematic situation, it is recommended to set the overlap ratio  $\alpha$  as large as possible when designing the viewing parameters, since it provides more robustness against larger variances in  $d'_{pupil}$ .

### III. CONCLUSION

The tradeoff relation between the design parameters of a pinhole-based NED, depending on various viewing parameters, has been analyzed. The distance  $D_{pp}$  from the pinhole array to the eye's lens decides the retinal angular resolution  $A_R$  and overlap ratio  $\alpha$ , which are inversely proportional. Though high retinal angular resolution is a key factor in enhancing the observed image quality, the overlap ratio is also important, to prevent the problem of empty spaces between retinal elemental images. Therefore, the robustness against variances in viewing conditions such as the real pupil diameter should be the first design parameter to be considered, after which enhancing the picture quality (the angular retinal resolution) of a NED will be meaningful.

### ACKNOWLEDGMENT

This research was supported by the Basic Science Research Program through the National Research Foundation

of Korea (NRF), funded by the Ministry of Education (2018R1D1A1B07049563); by the ITRC (Information Technology Research Center) support program (IITP-2020-2015-0-00448), supervised by the IITP (Institute for Information and communications Technology Promotion); and by an IITP grant funded by the Korean government (MSIP) (2019-0-01347, Development of Realistic Fire Training Content Technology to Help Simulate Fire Sites and Improve Command Capabilities).

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